

# Parameter selection for model updating based on the global sensitivity method

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**Abstract.** A new model-updating parameter selection method based on global sensitivity analysis is presented in this work. A specifically designed evaluation function is used for the probability that the sample fits the distribution of test data. In contrast to other parameter selection methods the test-data information is introduced to the parameter selection procedure. Global sensitivity analysis is performed and a set of composite indices for parameter selection is calculated. The parameters are selected based on the values of these composite indices. The method is validated using simulation data from a pin-jointed truss structure model. The cases of independent and correlated parameters are studied and the presented method is shown to be effective for both.

## 1. Introduction

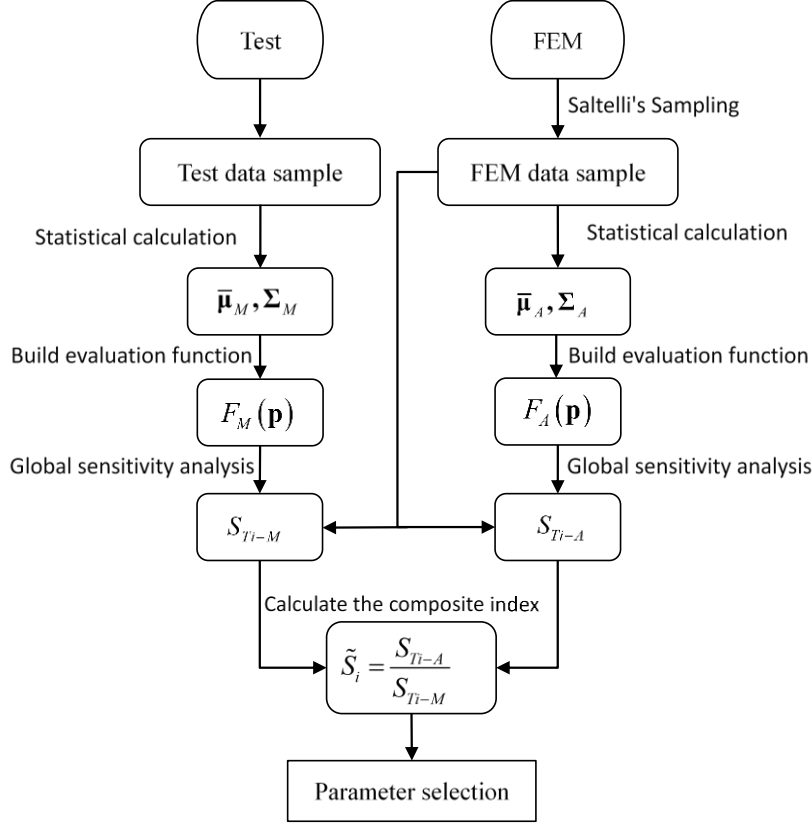
Finite element model updating [1], [2] is now a widely used technique in many industries, but the problem of selecting a suitable set of candidate updating parameters remains to be fully solved. There have been many studies carried out on this topic [3-5] often making use of local sensitivity data. These approaches are dependent upon initial parameter estimates and are not able to provide an assessment of the global influence of a candidate parameter.

Statistical methods were introduced to overcome this problem, including Bayesian evidence [6] and the F-test [7]. Global-sensitivity based methods are among the most popular [8], [9], but none of these involve the test data, which carries information on the modelling error. In [10] a local sensitivity-based technique was introduced which selects parameters that contribute most to the output test-data covariance terms, but it still needs an initial guess of the parameters and is limited by an assumption of uncorrelated parameters.

The purpose of the present research is to propose a global sensitivity-based parameter selection method, which uses both information from the model and from the test data. This is done by designing an evaluation function, with built-in test data, and based on the multivariate normal distribution probability density function (PDF). Then by using the global sensitivity method, the evaluation function is analysed and the parameters that have a higher contribution to the uncertainty of the model error are chosen as updating parameters. Effectiveness and robustness are validated in two numerical examples of a truss structure with independent parameters and correlated parameters.

## 2. Parameter selection by using global sensitivity

The parameter selection method is described by the flowchart shown in Figure 1 and details are provided in the following sections.



**Figure 1.** Flowchart of the method.

### 2.1. Building the evaluation function

Consider an analytical finite element model with input parameter vector  $\mathbf{p} = (p_1, p_2, \dots, p_n)^T$  and output modal frequencies  $\boldsymbol{\omega}_A = (\omega^1, \omega^2, \dots, \omega^k)_A^T$ , where  $n$  is the number of parameters to be selected and  $k$  is the number of output modal frequencies. The model can be described as a function,

$$\boldsymbol{\omega}_A = f_A(\mathbf{p}). \quad (1)$$

The model output is chosen here to be the set of modal natural frequencies, but could include mode shapes, frequency response functions or any other model output.

The corresponding test data measurement can be described as a random vector for each of  $k$  natural frequencies  $(\boldsymbol{\omega}_M^i)^T = (\omega_1^i, \omega_2^i, \dots, \omega_m^i)_M$ ;  $i = 1, 2, \dots, k$  obtained typically from tests carried out on  $m$  nominally identical structures. To show no preference for any particular output, the samples from finite element analysis and test are scaled with the mean of the test data  $\mu_M^i = E[\boldsymbol{\omega}_M^i]$  such that,

$$\mathbf{y}_A^i = \frac{\boldsymbol{\omega}_A^i}{\mu_M^i} = \frac{f_A^i(\mathbf{p})}{\mu_M^i}. \quad (2)$$

$$\mathbf{y}_M^i = \frac{\boldsymbol{\omega}_M^i}{\mu_M^i}. \quad (3)$$

Then the normalized mean  $\bar{\boldsymbol{\mu}}_M$  and the covariance matrix  $\boldsymbol{\Sigma}_M$  may be determined as,

$$\begin{aligned} \bar{\boldsymbol{\mu}}_M &= \mathbf{I} = (1, 1, \dots, 1)_n^T \\ \boldsymbol{\Sigma}_M &= E[(\mathbf{y}_M - \mathbf{I})(\mathbf{y}_M - \mathbf{I})^T] \end{aligned} \quad (4)$$

Similarly, by sampling from a multivariate normal distribution of the modelling parameters, and in preparation for a later requirement in §2.3, the mean  $\bar{\boldsymbol{\mu}}_A$  and the covariance matrix  $\boldsymbol{\Sigma}_A$  of the normalized finite element model output sample may be obtained by forward propagation.

Then, from (4) an evaluation function in the form of a log multivariate normal distribution PDF may be constructed as,

$$F_M(\mathbf{p}) = \log \text{PDF}_M(\mathbf{y}_A(\mathbf{p})) = \log\left(\frac{\exp(-\frac{1}{2}(\mathbf{y}_A(\mathbf{p}) - \mathbf{I})^T \boldsymbol{\Sigma}_M^{-1}(\mathbf{y}_A(\mathbf{p}) - \mathbf{I}))}{\sqrt{(2\pi)^k |\boldsymbol{\Sigma}_M|}}\right). \quad (5)$$

The log form of multivariate normal distribution PDF gives a higher gradient when the sample is far from the mean value. This can be helpful in preventing the sensitivity from becoming too small when the samples are far from the mean of the test data. The physical meaning of this evaluation function is the probability that the simulation result  $\mathbf{y}_A$  calculated with given parameter vector  $\mathbf{p}$  fits the distribution of the test data. Therefore, the parameters that should be updated may be selected by analysing the sensitivity of this evaluation function.

## 2.2. Global sensitivity analysis

There are many ways of calculating the global sensitivity indices including Fourier Amplitude Sensitivity Test (FAST), the method of Morris, the Derivative-based Global Sensitivity Measure (DGSM) and the Delta Moment-Independent Measure. Here Sobol's sensitivity analysis method (originally developed by Sobol [11] and improved by Saltelli [12]) is adopted for reasons of robustness and widespread usage. All these methods share the same concept of global sensitivity analysis, so any of them may be used in the research presented here.

In the present research, the parameters show an obviously high order effect brought by the interaction of parameters in the finite element analysis. Therefore, the total sensitivity indices will be mainly used. Fortunately, there is a powerful open source Python library (SALib [13]) which provides a convenient way to access the most popular global sensitivity methods [12] and is validated to be effective and efficient. The sampling procedure and the sensitivity evaluation were carried out using the functions available in this toolbox.

## 2.3. Eliminating the effect of the model

The evaluation function Eq. (5) gives the probability that a sample finite-element output falls within the distribution of the test data. But just as it contains the information from the test data, it also contains information from the finite element model itself, which will affect the selection and must be removed. It has been found intuitively that this can be achieved by the use of a baseline (or reference) function in the same form as the evaluation function, but with finite element data as  $F_A(\mathbf{p})$ .

Then, to eliminate unwanted model effects a new composite index may be created by dividing the baseline total sensitivity index by the same index from the evaluation function,

$$\tilde{S}_i = \frac{S_{Ti-A}}{S_{Ti-M}}. \quad (5)$$

where  $S_{Ti-A}$  and  $S_{Ti-M}$  denote the total global sensitivity of the baseline function and of the evaluation function respectively. Then  $\tilde{S}_i$  is the composite index that is used for parameter selection. From the geometric characteristics of multivariate normal distribution, the curve tends to be flat when the variance is high. This means that those parameters with high uncertainty will tend to have low sensitivity. This is why the baseline-function sensitivity appears in the numerator and the test function in the denominator, with the result that the parameters to be selected appear with higher values of the composite index than the other parameters.

### 3. Numerical case study

#### 3.1. Pin-joined truss

As shown in Fig.2, a pin-joined truss structure of overall dimensions 5m×1m and composed of 21 bar elements, will be used as a simulated numerical case study.

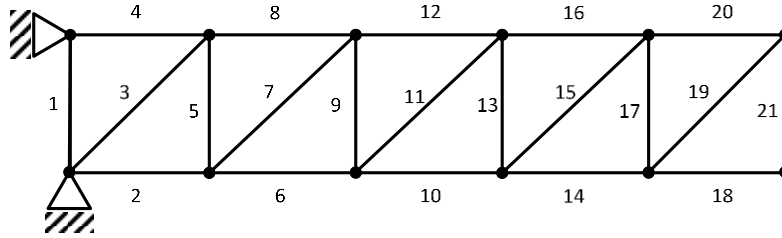


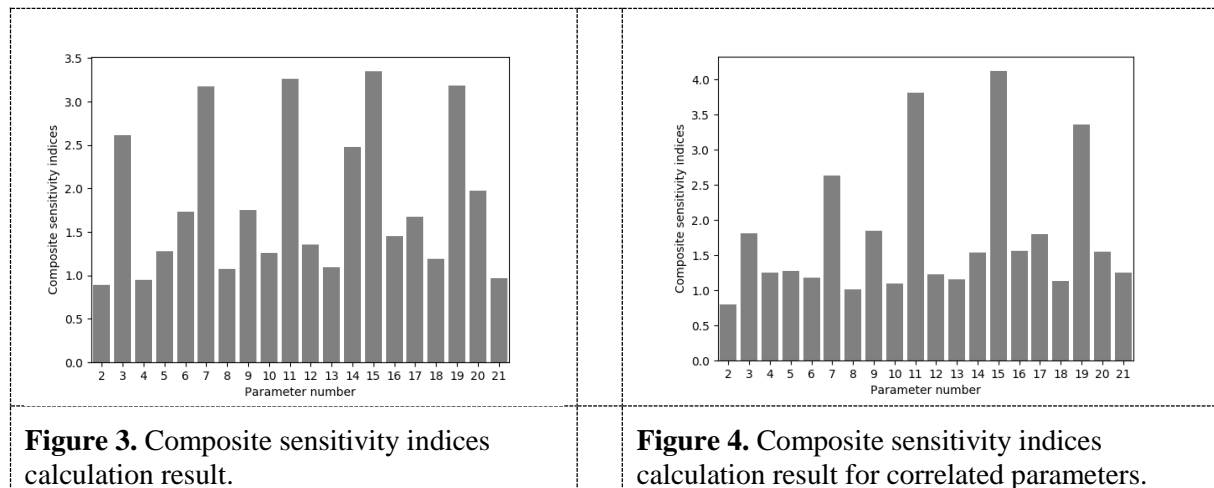
Figure 2. Pin-joined truss.

Each bar element is randomised and considered as a candidate for updating. Therefore, the whole structure is an uncertain system with 21 randomized parameters. Then, in what follows, the mean and standard deviation values of chosen parameters are changed to simulate highly erroneous parameters in the structure. Modal frequency data are used as the nominal measured output and using the parameter selection method presented in §2, the parameters that contribute most to the uncertainty of the structure will become apparent.

#### 3.2. Selection result for independent and correlated parameter cases

The first case is when the parameters to be selected are independent of each other. Then, using numbering of elements as in Figure 3, the diagonal bars listed as [3,7,11,15,19] are chosen as target parameters. The mean and standard deviation values of parameters in this list are changed when generating the nominal test data.

Then in the correlated parameters case, we used the same list of target parameters. The only difference is when generating the nominal test data, the parameters in the list are correlated with each other. The result of the parameters selection is presented in Fig.3 and Fig.4. respectively. We can see in both cases, the parameters in the target list are correctly selected. But in the correlated case, the result is still affected by the correlation of the parameters. As shown in Fig.4, parameters  $p_3$  and  $p_7$  have a very strong correlation, which makes them have lower index values than other parameters. Also, as for the effect of correlation, parameter  $p_9$  may also be mistakenly selected. This is not exactly what we want, but for most model updating problems, selecting a limited extra number of parameters than needed will not have much effect on the updating result. Thus, correlated parameters may also be selected by the method presented in this paper.



#### 4. Conclusions

In this paper, a parameter selection method for stochastic model updating is presented. The method is based global sensitivity analysis and a new evaluation function is developed that makes use of the probability that a sample from finite element analysis fits the distribution of the test data. Two simulation cases studies were presented with parameter selection carried out on a pin-jointed truss structure. In the first study, the uncertain parameters were taken to be independent and in the second the parameters were correlated. All stiffness parameters in the structure were randomized with chosen mean values and standard deviations, but only a subset of these were erroneous. Excellent results were obtained in both cases. When the parameters were independent the selection was perfect, and in the case of correlated parameters, it was found that, in addition to all the erroneous parameters, the correlation resulted in one additional parameter being falsely selected.

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#### References

- [1] Mottershead J E and Friswell M I 1993 Model updating in structural dynamics: A survey *J. Sound Vib.* **167** 347–75
- [2] Friswell M I and Mottershead J E 1995 *Finite element model updating in structural dynamics* vol 38
- [3] Friswell M I, Mottershead J E and Ahmadian H 1998 Combining Subset Selection and Parameter Constraints in Model Updating *J. Vib. Acoust.* **120** 854
- [4] Mottershead J E, Mares C, Friswell M I and James S 2000 Selection and updating of parameters for an aluminum space-frame model *Mech. Syst. Signal Process.* **14** 923–44
- [5] Abu Husain N, Haddad Khodaparast H and Ouyang H 2012 Parameter selection and stochastic model updating using perturbation methods with parameter weighting matrix assignment *Mech. Syst. Signal Process.* **32** 135–52
- [6] Mthembu L, Marwala T, Friswell M I and Adhikari S 2011 Model selection in finite element model updating using the Bayesian evidence statistic *Mech. Syst. Signal Process.* **25** 2399–412
- [7] Fang S-E and Perera R 2011 Damage identification by response surface based model updating using D-optimal design *Mech. Syst. Signal Process.* **25** 717–33
- [8] Saltelli A, Ratto M, Andres T, Campolongo F, Cariboni J, Gatelli D, Saisana M and Tarantola S 2008 *Global Sensitivity Analysis. The Primer*
- [9] Sudret B 2008 Global sensitivity analysis using polynomial chaos expansions *Reliab. Eng. Syst. Saf.* **93** 964–79

- [10] Silva T A N, Maia N M M, Link M and Mottershead J E 2016 Parameter selection and covariance updating *Mech. Syst. Signal Process.* **70–71** 269–83
- [11] Sobol I M 2001 Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates *Math. Comput. Simul.* **55** 271–80
- [12] Saltelli A, Annoni P, Azzini I, Campolongo F, Ratto M and Tarantola S 2010 Variance based sensitivity analysis of model output. Design and estimator for the total sensitivity index *Comput. Phys. Commun.* **181** 259–70
- [13] Herman J and Usher W 2017 SALib: An open-source Python library for Sensitivity Analysis *J. Open Source Softw.* **2**